

EXACT VS. GAUSS-SEIDEL NUMERICAL SOLUTIONS OF THE NON-LTE RADIATION TRANSFER PROBLEM

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Abstract. Although published in 1995, the Gauss-Seidel method for solving the non-LTE radiative transfer problem has deserved too little attention in the astrophysical community yet. Further tests of the performances and of the accuracy of the numerical scheme are provided.

1 Introduction

Fast and *accurate* numerical schemes for the solution of the non-LTE radiation transfer (RT) problem still need to be pushed further, in order to be able to deal with increasingly complex models (e.g., multi-dimensional geometry, multi-level atoms, polarisation...). Hereafter, we present new numerical tests against *exact* solutions of the Gauss-Seidel (GS) and SOR methods initially proposed by Trujillo Bueno & Fabiani Bendicho (1995), and inspired by the classical GS iterative method in numerical analysis (which can be modified into SOR methods when an overcorrection is made as compared to GS).

Very few tests, indeed, are available for the validation of any new numerical scheme dealing with the resolution of the non-LTE radiative transfer problem. The usual one comes from the computation of numerical solutions under the Eddington approximation i.e., adopting a very coarse angular quadrature such as $\mu = \pm 1/\sqrt{3}$ (also known as the “two-stream” approximation; see §4.3.1. in Rutten 2003). In such a case, analytical solutions – of a “reduced” RT problem though – are available for numerical solutions to be checked against. However this test may lead to an *erroneous* estimation of the accuracy of the numerical scheme, as pointed out in Chevallier et al. (2003, see §5. therein).

2 Numerical tests and discussion

The GS/SOR numerical schemes are better implemented when one adopts the “short characteristics” (SC) approach for the so-called formal solution of the RT

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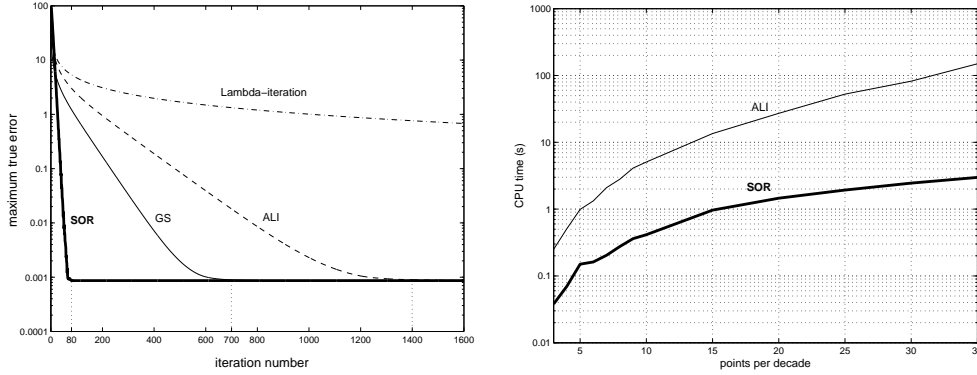


Fig. 1. *Left:* evolution of the maximum true error vs. the number of iterations (20 grid-points per decade) *Right:* CPU-time for, respectively, the ALI and the GS/SOR numerical schemes vs. the number of points per decade (for a 1D finite slab of optical thickness 1000 and a collisional destruction probability 10^{-4}).

equation (see Auer & Paletou 1994). Chevallier et al. (2003) showed how the accuracy of an Accelerated Λ -Iteration (with a monotonic parabolic SC formal solver) numerical code's solutions do scale with the refinement of both spatial and angular grids, by comparing the latter to very high-accuracy analytical solutions given by the ARTY code (Chevallier & Rutily 2004).

We computed GS/SOR solutions for a grid of 1D finite slab, two-level atom (in CRD) models that we compared to ARTY reference solutions: the rate of convergence for various numerical schemes is displayed in Fig. 1 (left). Our main conclusions are that: (a) as expected, the *accuracy* of the GS/SOR code is identical to the one of the ALI-SC-based code, (b) the numerical “overcost” for GS iterations (due to a modified formal solver) is *negligible* but, (c) that the gain in computing time with GS/SOR is *very significant*. As shown in Figs. 1, for high-order quadratures, which are *absolutely needed* in order to keep the accuracy of the ALI-SC method better than 1% (see Chevallier et al. 2003), the gain in CPU-time provided by the GS/SOR scheme can be as large as a factor of 50! And even with standard acceleration techniques for ALI or GS, GS/SOR remains the fastest algorithm (see Trujillo Bueno & Fabiani Bendicho 1995).

We feel that this fully justifies to consider *very seriously* GS/SOR schemes for future radiative modelling codes (note also that GS/SOR have already been generalized to complex models). It is particularly important for the development of those diagnosis tools required by major projects such as GAIA or HERSCHEL, for instance.

References

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